A Comment on "Dynamics of Weak First Order Phase Transitions"

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Nucleation and spinodal decomposition, as mechanisms for first-order transitions, have been studied for many years. A pertinent issue is whether or not there is a sharp distinction between these two processes. In Ref. [1], M. Gleiser analyzes a lattice model with one free parameter α which is tuned to control the strength of a first–order transition. He argues that there exists a critical value α_c distinguishing between strong and weak first–order transitions and suggests that only for $\alpha > \alpha_c$ does phase transformation proceed by nucleation. We believe that the numerical results in this work are correct, but will argue against the interpretation of the theory at α_c as a boundary between weak and strong phase transitions.

The phenomenology of phase transitions can be described by the Landau–Ginzburg Hamiltonian

$$\frac{H}{\theta} = \frac{1}{\theta}\left(\frac{1}{2}(\nabla\phi)^2 \pm m^2\phi^2 + \frac{\lambda}{4}\phi^4 + \beta(t)\phi + \ldots\right). \tag{1}$$

As the system is cooled, $\beta(t)$ changes sign, the minimum of the free–energy shifts discontinuously, and phase transformation occurs. In Ref. [1], the above Hamiltonian, with $\beta(t) \equiv 0$ and a positive coefficient of ϕ^2, is adopted as a *lattice* Hamiltonian. This free energy has two degenerate minima; ϕ is placed throughout the lattice in the left–most minima and its relaxation (with second order Langevin dynamics) is studied; α controls the height of the hump between the two minima. At a certain non–zero value of α, the relaxation dynamics becomes critical. The size of the potential hump is interpreted as a measure of the strength of a first–order transition that would occur under cooling, implemented by a time–dependent parity–odd perturbation of the Hamiltonian. For $\alpha \gg \alpha_c$, the bubble free energy is large and nucleation should adequately describe the relaxation. It is then inferred that a different relaxation process may operate when $\alpha < \alpha_c$.

This description is misleading because it depends on the shape of the lattice (bare) potential. The physics of the phase transformation is modeled by the *coarse–grained* potential, related to the bare potential by renormalization. Criticality of the relaxation process depends only on the long–time equilibrium behavior. Thus, we only need the renormalization corrections for a classical statistical mechanical model at finite temperature θ, or equivalently, a quantum field theory with $\hbar = 1/\theta$. Explicitly, consider the one-loop correction to the quadratic term, which depends on the inverse lattice spacing Λ as

$$H_{ct} = \frac{1}{4\pi} 3\lambda\theta \ln\left(\frac{\Lambda^2}{\mu^2(m,\lambda\theta)}\right)\ \phi^2; \tag{2}$$

the finite part of the counterterm, μ, is determined by imposing renormalization conditions. The leading order Λ dependent piece reduces the barrier in the coarse–grained free energy by an additive correction. We anticipate that at some value of α, presumably α_c, the quadratic term and bump in the continuum effective potential vanish. This corresponds to the second–order phase transition in the ϕ^4 system, which is in the Ising universality class.

Indeed, with this correspondence in mind, simulations of the lattice Hamiltonian of Ref. [1] have been previously performed. In Ref. [2], Monte Carlo simulations are used to extract Ising critical exponents from this model. In Ref. [3], Monte Carlo and Langevin simulations are used to obtain the critical line of the lattice ϕ^4 model. The coupling χ of Ref. [3] is exactly the quantity $\lambda\theta_c$ of Ref. [1]; the coupling θ^{TC} of Ref. [3], which we distinguish by a superscript TC, equals $(\theta_c^2 - 1)/2$ in Ref. [1], with $\theta_c = (1 - 2\alpha^2/9\lambda)^{-1/2}$. In Ref. [1], criticality occurs at $(\lambda, \alpha) = (.1, .36)$. This corresponds to $(\chi, \theta^{TC}) = (.119, .202)$, which lies within statistical error on the critical line presented in Table II and Figure 8 of Ref. [3]. We have repeated these numerical simulations, using Langevin dynamics as in Ref. [3]. We find results consistent with those above. Thus the critical behavior in Ref. [1] is in the Ising universality class,

with vanishing coarse–grained barrier energy. It is not appropriately described in the continuum limit as a boundary between large and small barrier energies.

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